Amplificateurs Opérationnels (AO2)
Réponses fréquentielle et conception de Filtre

Systèmes électriques et électroniques
Adil KOUKAB

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- Réponse fréquentielle d'un montage à AO
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  - Exemples
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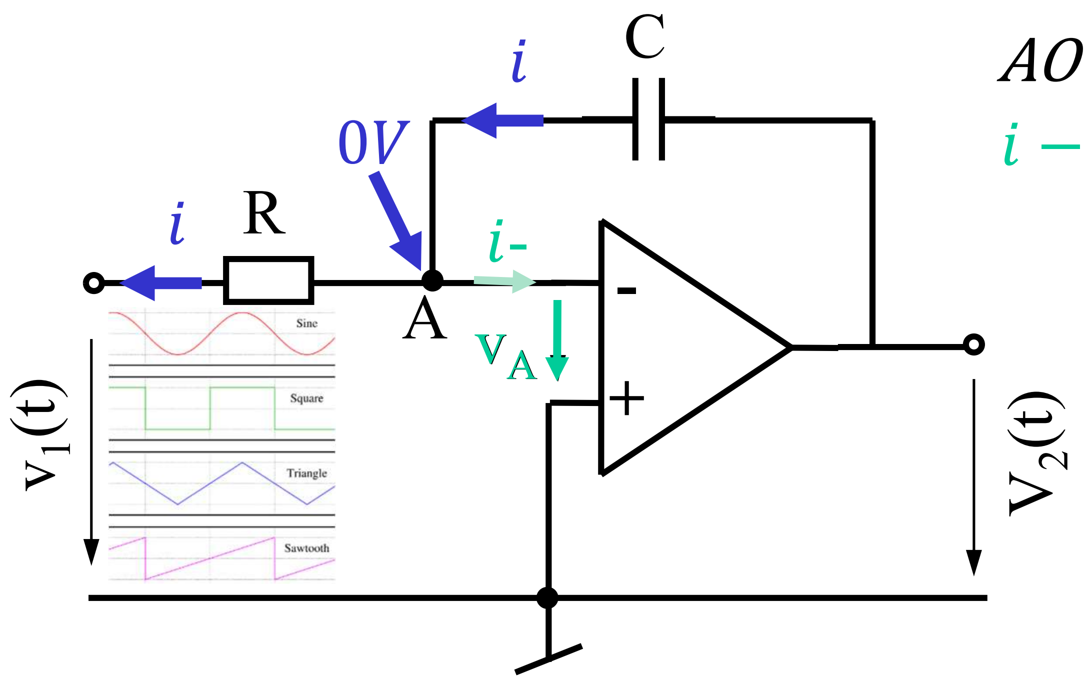
  - Topologie Générale
  - Calcul d'impédance de circuits R, C, RC, …
  - Conception de filtres:
    - d'ordre 1
    - d'ordre >1
  - Exemples

Montage intégrateur

Domaine temporel ($v_1(t)$ *quelconque*)



AO idéal + Réaction Négative

$i- = 0 \quad + \quad \Delta V = -V_A = 0$

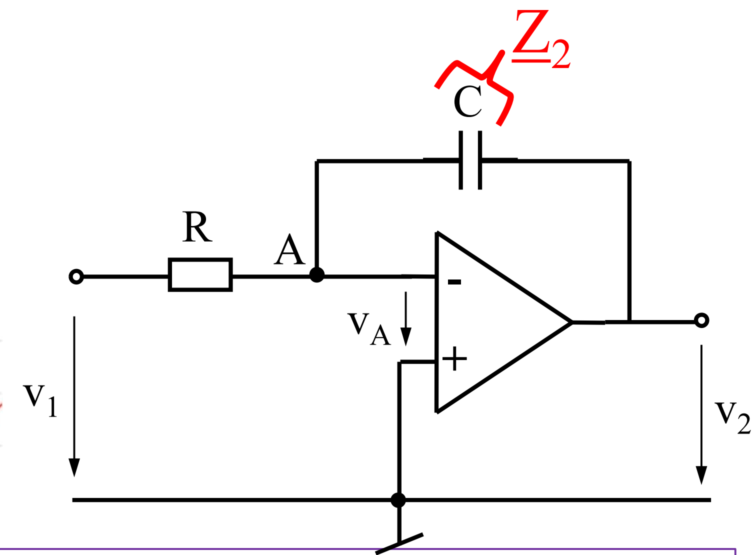
$$i = C\frac{d(V_2 - V_A)}{dt} = C\frac{dV_2}{dt}$$

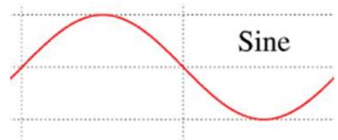
$$i = \frac{V_A - V_1}{R} = -\frac{V_1}{R}$$

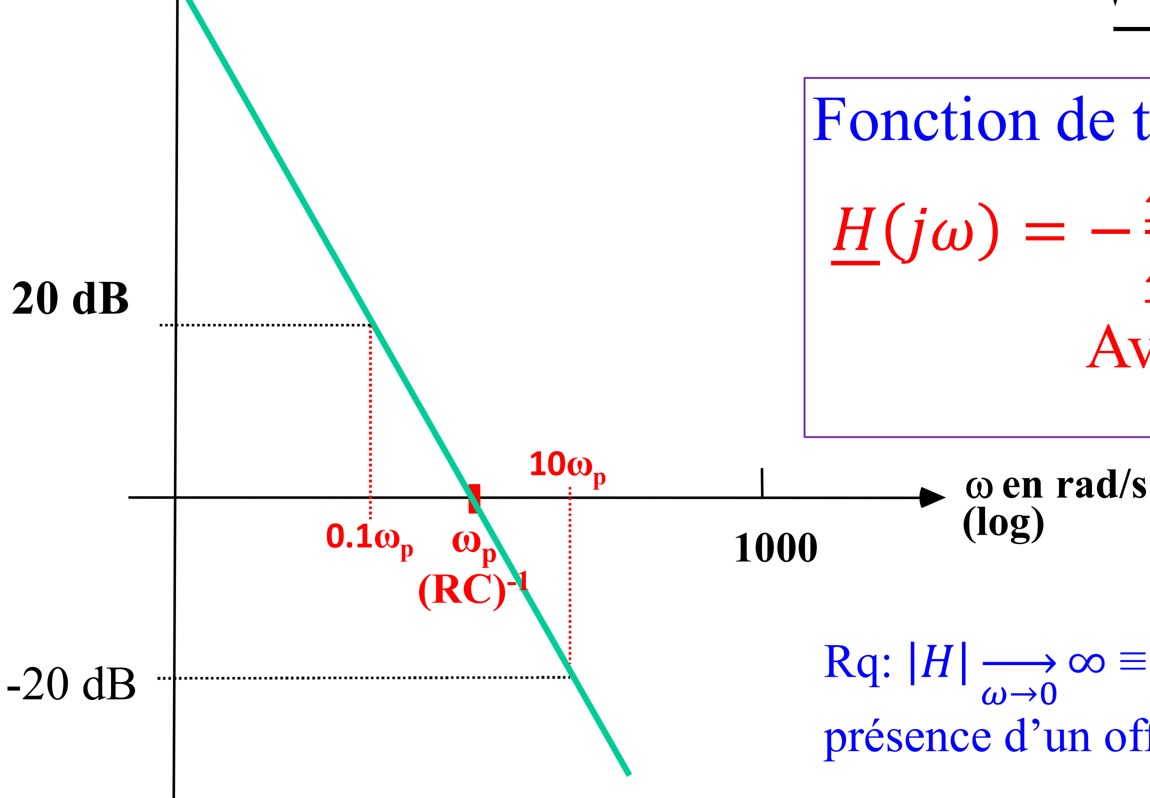
$$\rightarrow \frac{dV_2}{dt} = -\frac{V_1}{RC}$$

$$V_2(t) = -\frac{1}{RC}\int_0^t V_1\,dt + V_2(0)$$

Réponse Harmonique d'un intégrateur







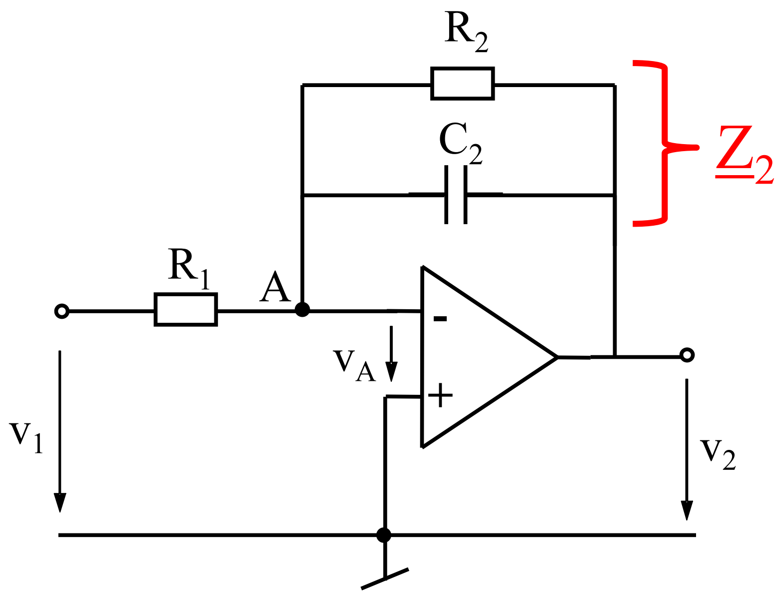
Fonction de transfert:

$$\underline{H}(j\omega) = -\frac{\underline{Z}_2}{\underline{Z}_1} = -\frac{1/jC\omega}{R} = -\frac{1}{j\omega/\omega_p}$$

Avec $\omega_p = (RC)^{-1}$

Rq: $|H| \xrightarrow[\omega \to 0]{} \infty$ ≡ Saturation de la sortie en cas de présence d'un offset (V_{1dc}) à l'entrée

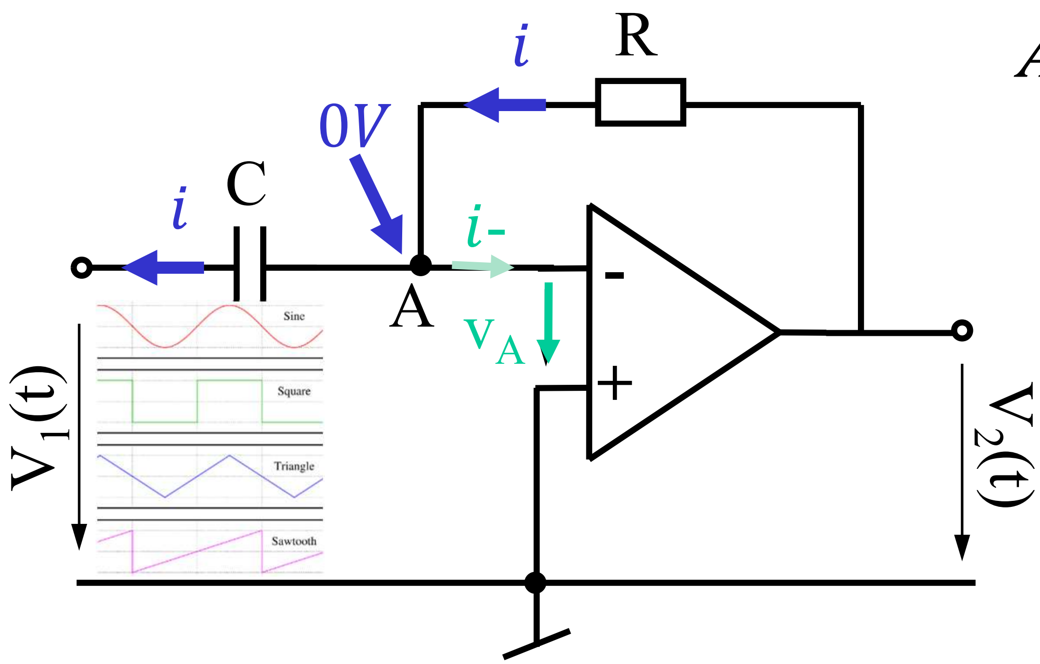
Exemple 2: Réponse fréquentielle d’un montage à AO



Etablir la fonction de transfert et tracer son diagramme de Bode en amplitude et en phase.
Identifier la zone d’amplification et la zone d’intégration.

Montage différentiateur

Domaine temporel ($v_1(t)$ *quelconque*)



AO idéal + Réaction Négative

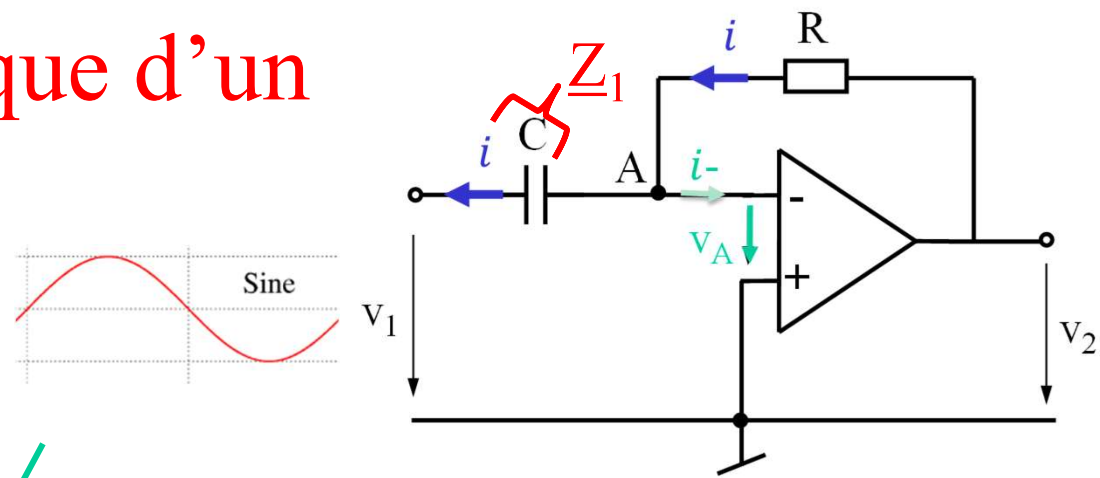
$i- = 0 \quad + \quad \Delta V = -V_A = 0$

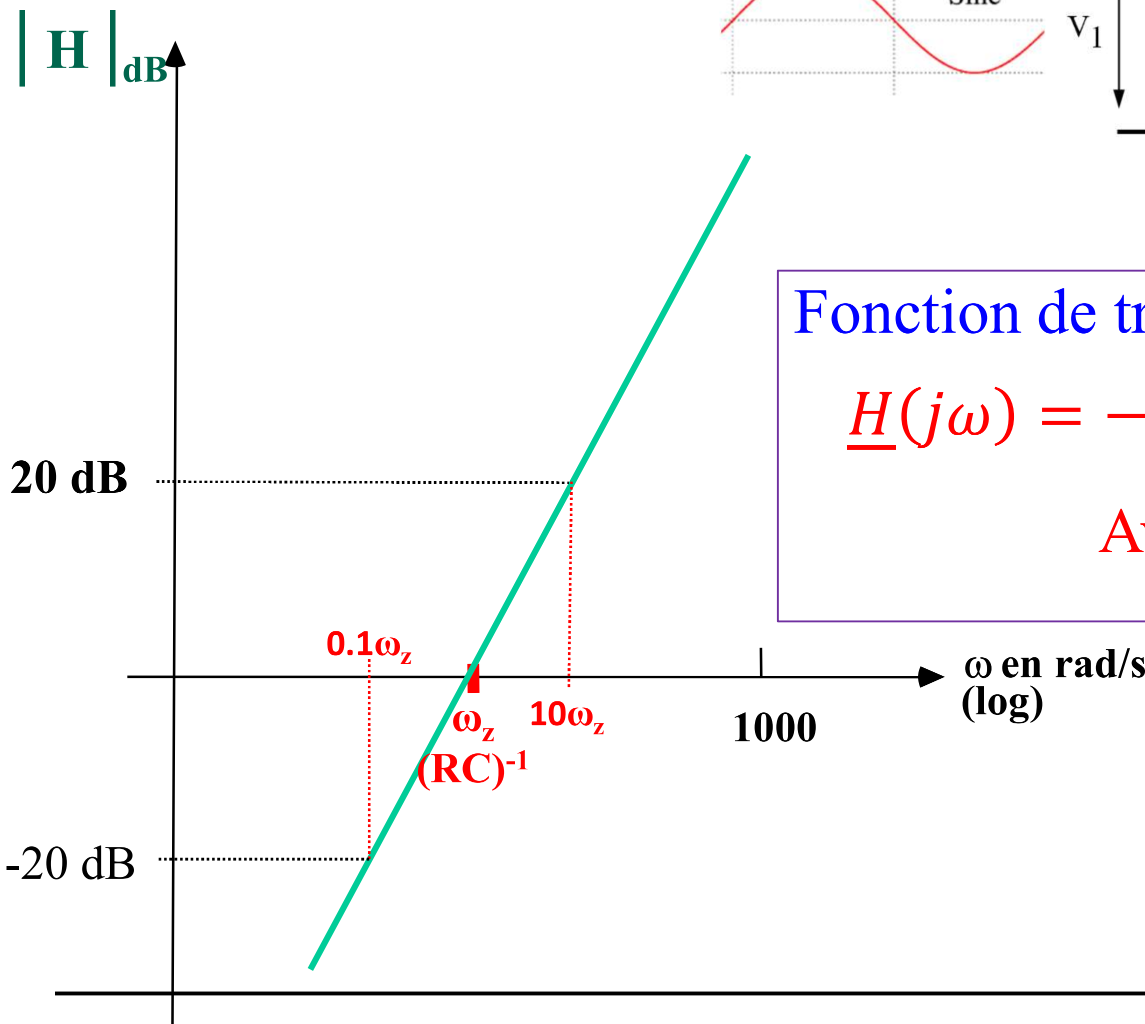
$$i = \frac{V_2}{R}$$

$$i = C\frac{d(V_A - V_1)}{dt} = -C\frac{dV_1}{dt}$$

$$V_2 = -RC\frac{dV_1}{dt}$$

Réponse Harmonique d'un différentiateur



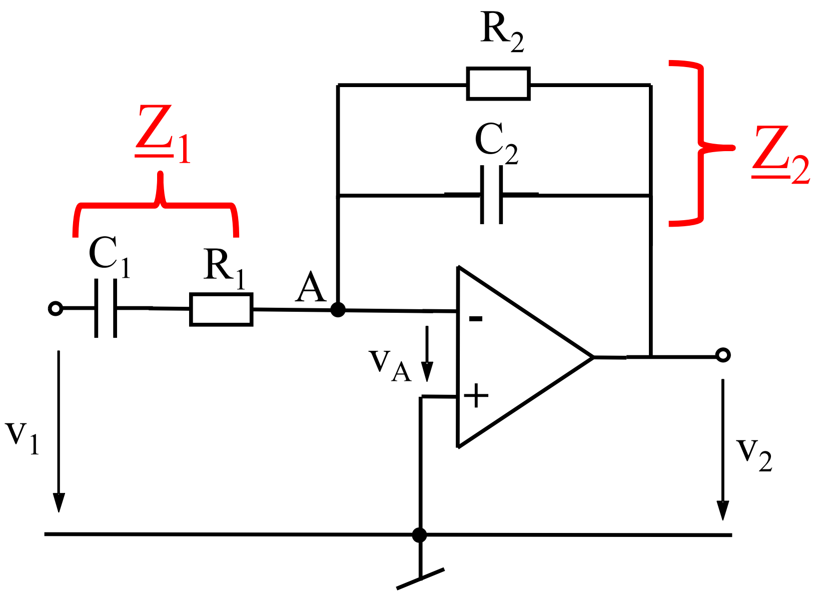


Fonction de transfert:

$$\underline{H}(j\omega) = -\frac{\underline{Z_2}}{\underline{Z_1}} = -j\omega RC = -j\omega/\omega_z$$

Avec $\omega_z = (RC)^{-1}$

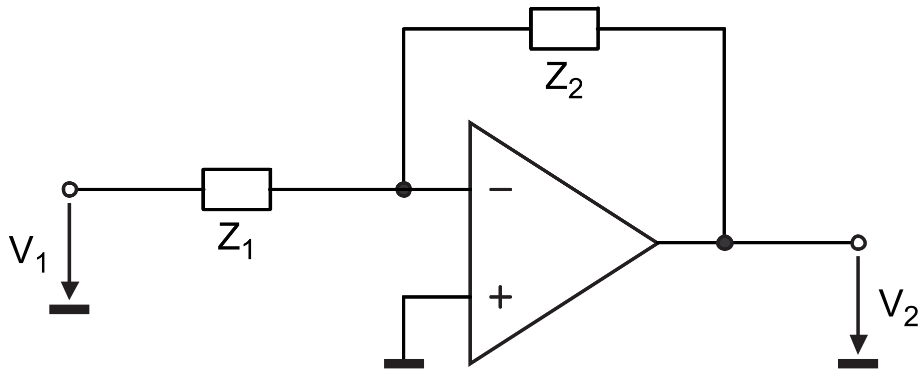
Exemple 2: Réponse fréquentielle d'un montage à AO



Etablir la fonction de transfert et tracer son diagramme de Bode en amplitude (supposer que $R_1 = 10\ R$ et $R_1C_1 = 10\ R_2C_2$)

Exemple de Conception de Filtres Actifs à base d'AO

Conception: Filtres d'ordre 1



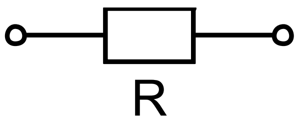
Circuit générique

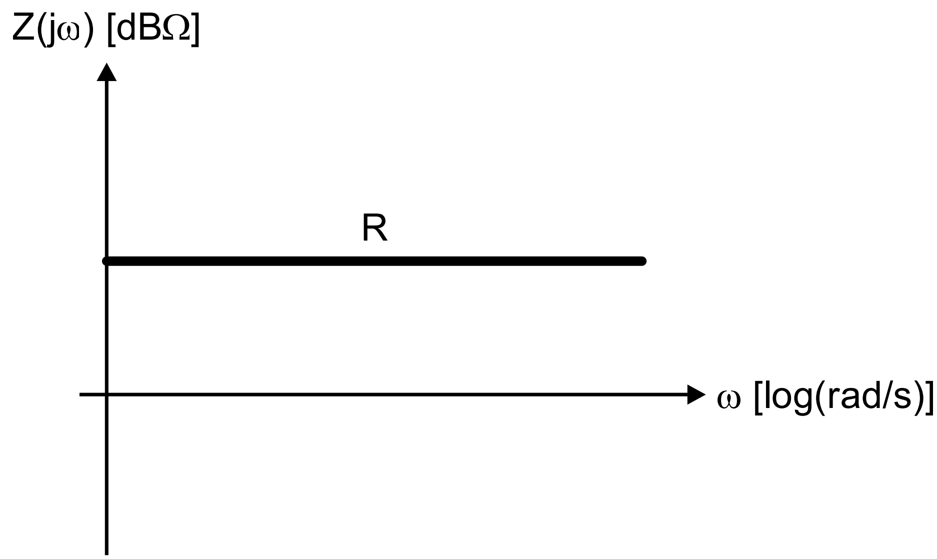
$$\underline{H}(j\omega) = \frac{\underline{v}_2}{\underline{v}_1} = -\frac{\underline{Z}_2}{\underline{Z}_1}$$

$$|\boldsymbol{H}|_{dB} = \underbrace{|\boldsymbol{Z_2}|_{dB\Omega}}_{\text{Inchangée}} - \underbrace{|\boldsymbol{Z_1}|_{dB\Omega}}_{\text{Inversée}}$$

- Choisir une structure pour le numérateur ($\underline{Z}_2$) et une pour le dénominateur ($\underline{Z}_1$)
- Les diagrammes de Bode correspondants se soustraient (celui du dénominateur $\underline{Z}_1$ doit être inversée verticalement, symétrie / axe des ω)

Impédance d'une résistance

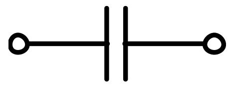




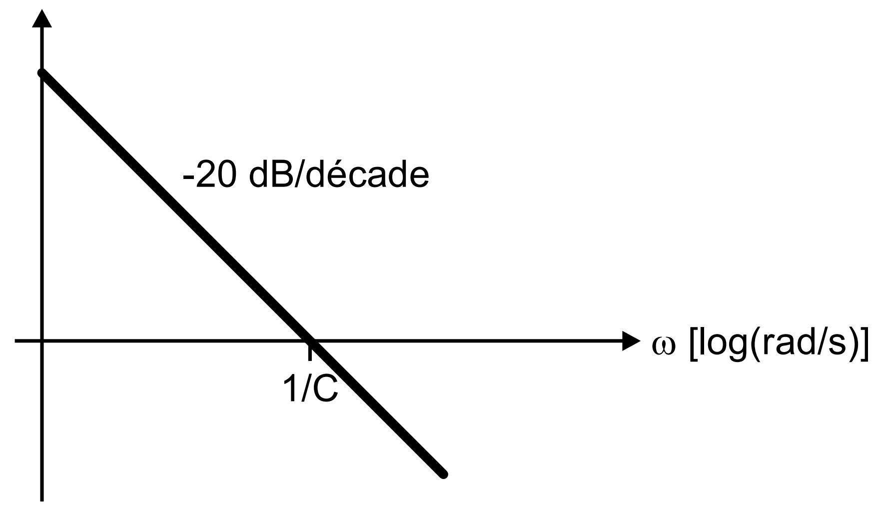
$$\underline{Z}_R(j\omega) = R$$

	f = 0 (DC)	f = ∞
Z =	R	R

Impédance d'une capacité

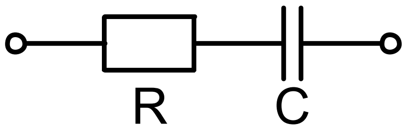


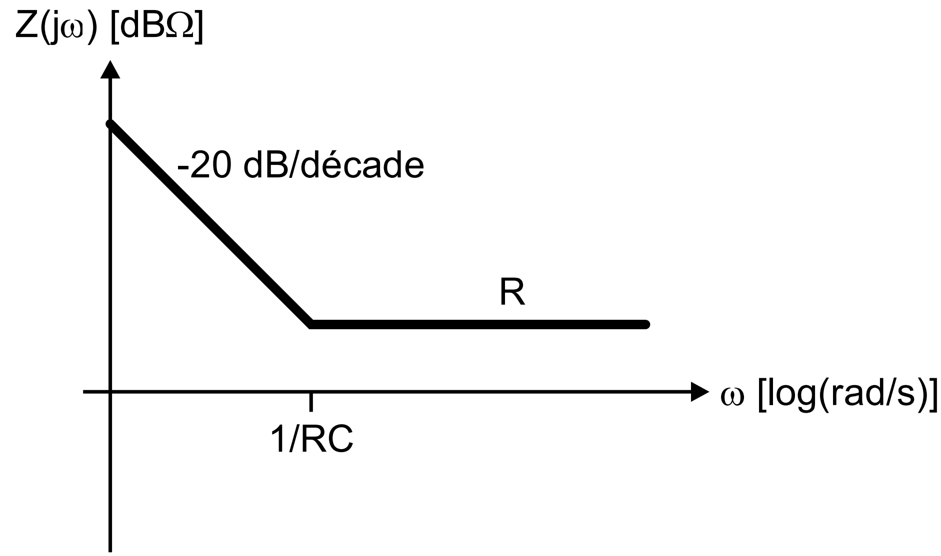
$$\underline{Z}_C(j\omega) = \frac{1}{j\omega C}$$



	f = 0 (DC)	f = ∞
Z =	∞	0 (-∞ dBΩ)

Montage RC série

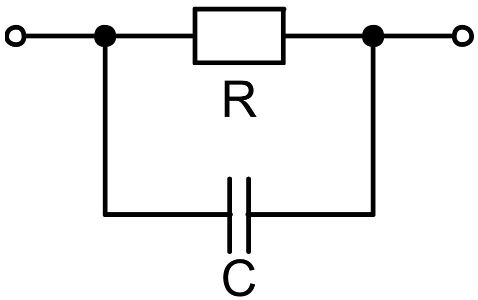




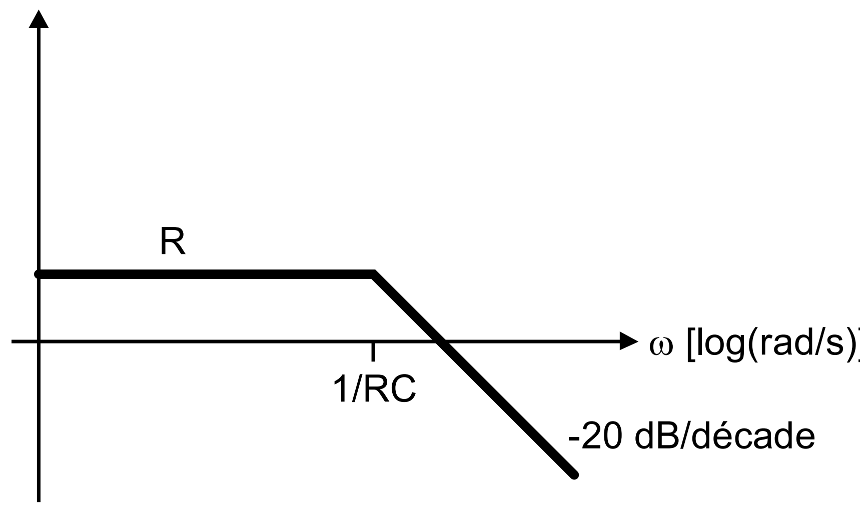
$$\underline{Z}(j\omega) = \frac{1}{j\omega C} + R = R\,\frac{1 + j\omega RC}{j\omega CR}$$

	f = 0 (DC)	f = ∞
Z =	∞	R

Montage RC parallèle

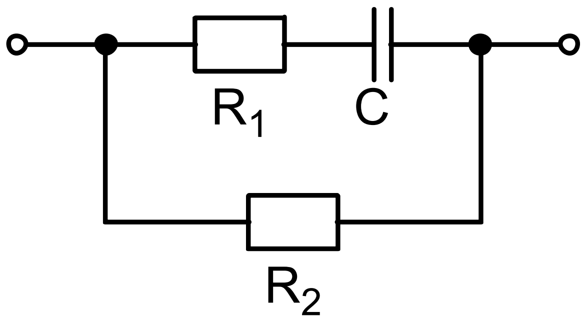


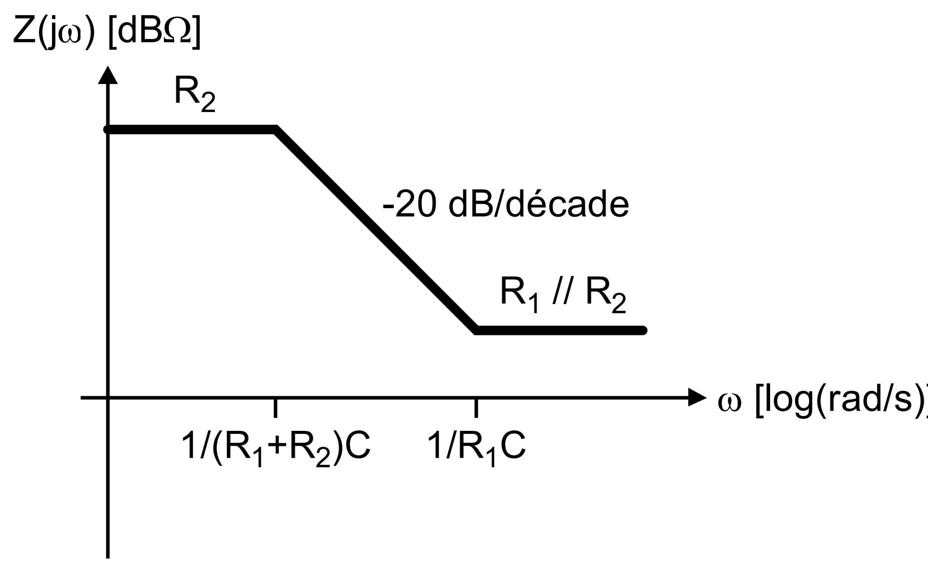
$$\underline{Z}(j\omega) = \frac{\frac{1}{j\omega C}\cdot R}{\frac{1}{j\omega C} + R} = \frac{R}{1 + j\omega RC}$$



	f = 0 (DC)	f = ∞
Z =	R	0 (-∞ dBΩ)

Montage R^2C



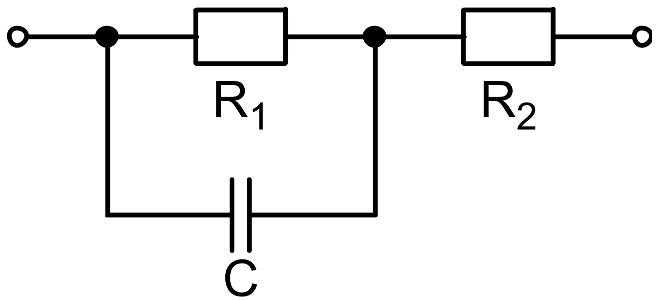


$$\underline{Z}(j\omega) = \frac{\dfrac{1 + j\omega R_1 C}{j\omega C} \cdot R_2}{\dfrac{1 + j\omega R_1 C}{j\omega C} + R_2}$$

$$= R_2 \frac{1 + j\omega R_1 C}{1 + j\omega (R_1 + R_2) C}$$

	$f = 0$ (DC)	$f = \infty$
$Z =$	R_2	$R_1 // R_2$

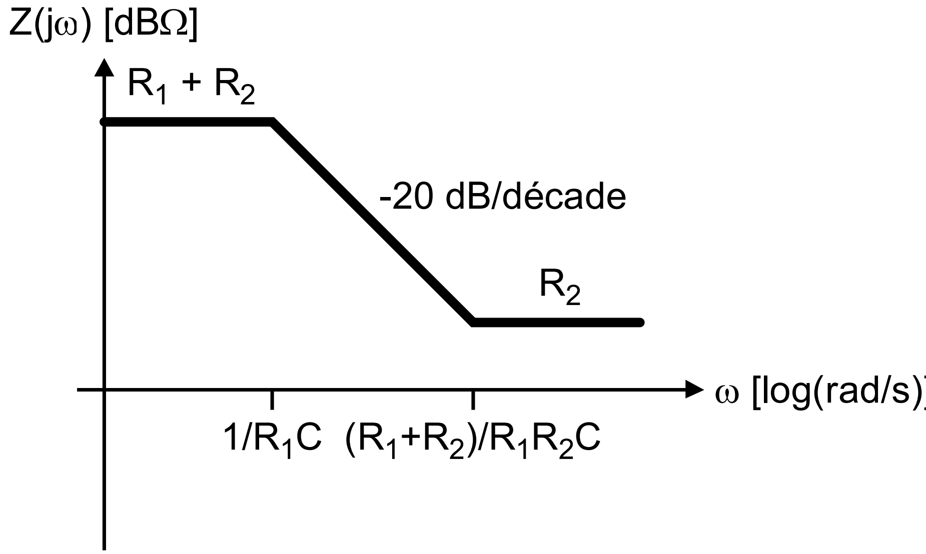
Montage R^2C



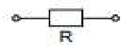
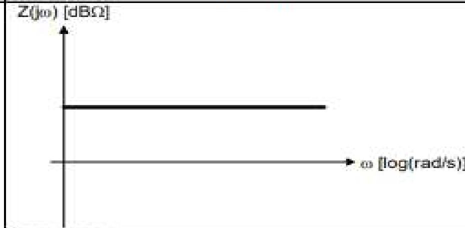
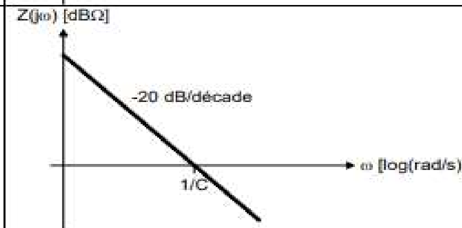
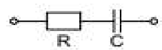
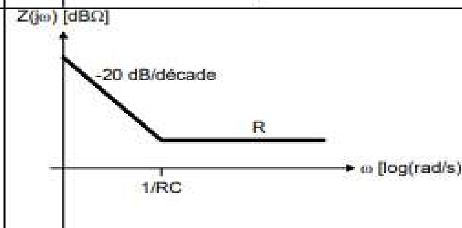
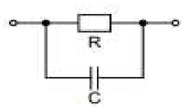
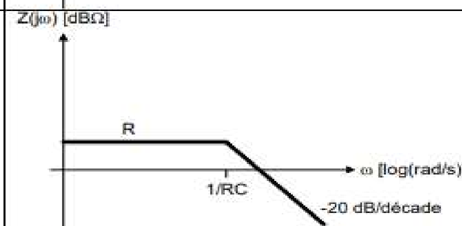
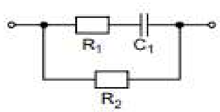
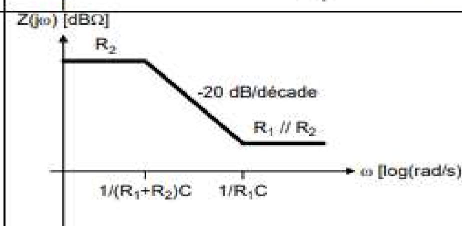
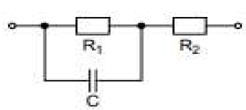
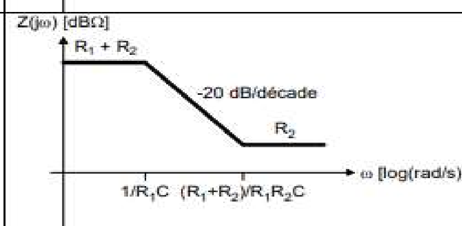
$$\underline{Z}(j\omega) = \frac{R_1}{1 + j\omega R_1 C} + R_2$$

$$= \frac{R_1 + R_2 + j\omega R_1 R_2 C}{1 + j\omega R_1 C}$$

$$= (R_1 + R_2)\frac{1 + j\omega \frac{R_1 R_2}{R_1 + R_2} C}{1 + j\omega R_1 C}$$



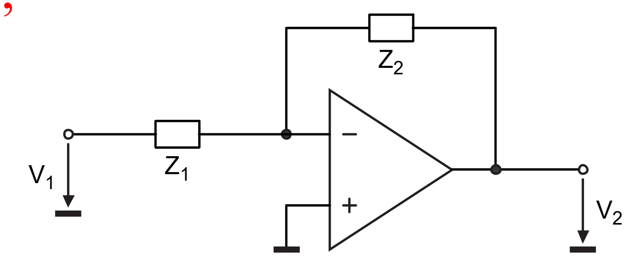
	$f = 0$ (DC)	$f = \infty$
$Z =$	$R_1 + R_2$	R_2

Circuit	Impédance $\underline{Z}(j\omega)$ — Equation	Impédance $\underline{Z}(j\omega)$ — Fonction
	R	
	$\dfrac{1}{j\omega C}$	
	$\dfrac{1+j\omega RC}{j\omega C}$	
	$\dfrac{R}{1+j\omega RC}$	
	$R_2\dfrac{1+j\omega R_1 C}{1+j\omega(R_1+R_2)C}$	
	$(R_1+R_2)\dfrac{1+j\omega\dfrac{R_1R_2}{R_1+R_2}C}{1+j\omega R_1 C}$	

Méthodologie de conception

I. A partir de la Forme du Diagramme de Bode,

- On conçoit la structure du Filtre



- Choix des impédances $\underline{Z}_1$ & $\underline{Z}_2$ dans le formulaire:

  Rq: $\underline{Z}_1$ au dénominateur de $\underline{H}$ → Son Bode est inversé verticalement, symétrie / axe des ω)

$$\underline{H}(j\omega) = \frac{\underline{v}_2}{\underline{v}_1} = -\frac{\underline{Z}_2}{\underline{Z}_1}$$

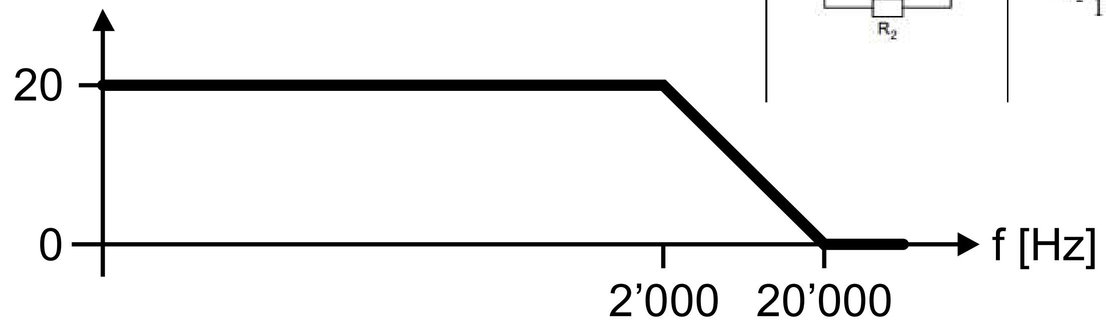
$$|\boldsymbol{H}|_{dB} = |\boldsymbol{Z_2}|_{dB\Omega} - |\boldsymbol{Z_1}|_{dB\Omega}$$

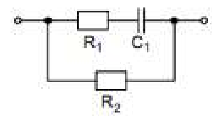
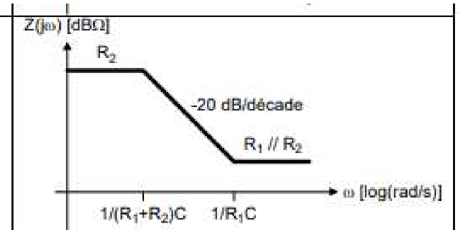
II. A partir des fréquences de coupures (pôles et zéros) et des gains,

- On dimensionne les composants $\underline{Z}_1$ & $\underline{Z}_2$

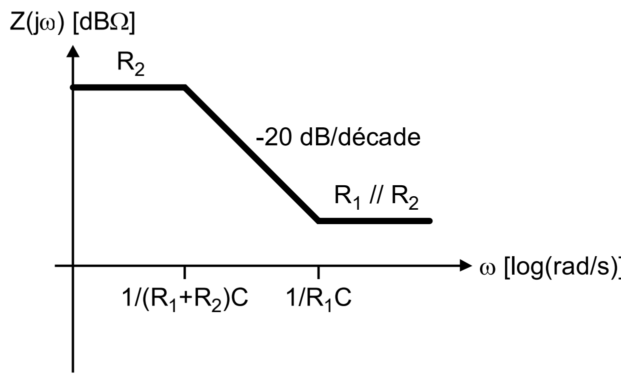
  → calcule les valeurs (R_i, C_i).

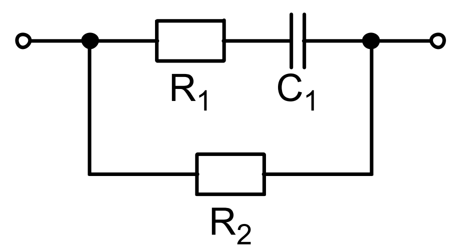
Example-conception 1



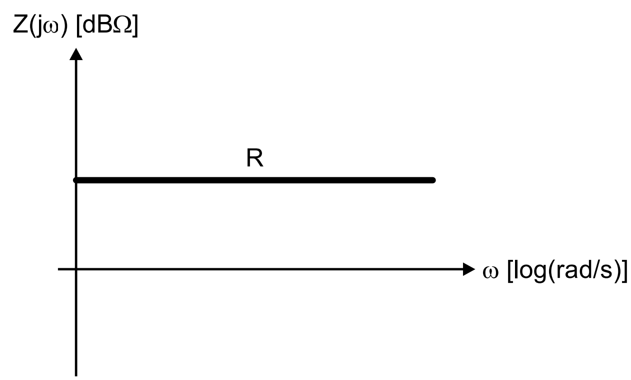
	$R_2 \frac{1 + j\omega R_1 C}{1 + j\omega (R_1 + R_2) C}$	

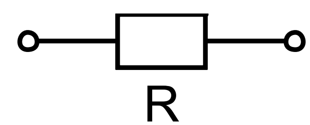
Choix des impédances et $\underline{H}(j\omega)$



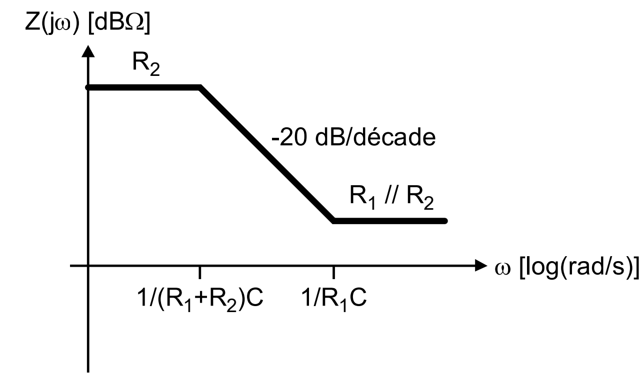


$$\left|\underline{H}(j\omega)\right| = \left|-\frac{\underline{Z}_2}{\underline{Z}_1}\right| =$$





Fonction de transfert



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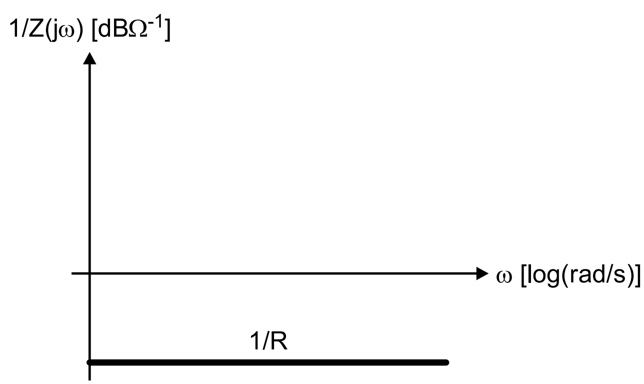
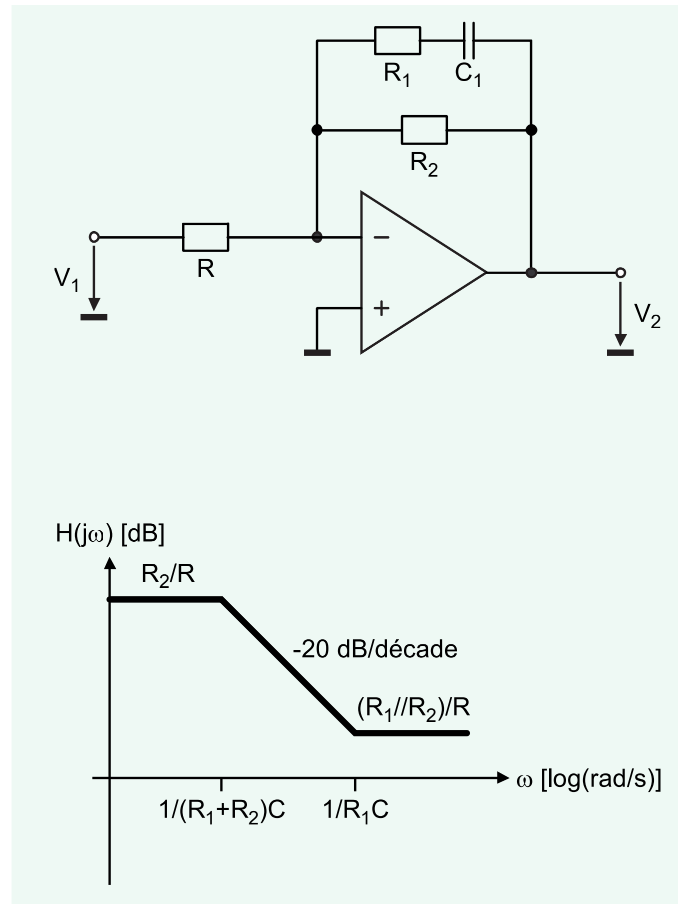
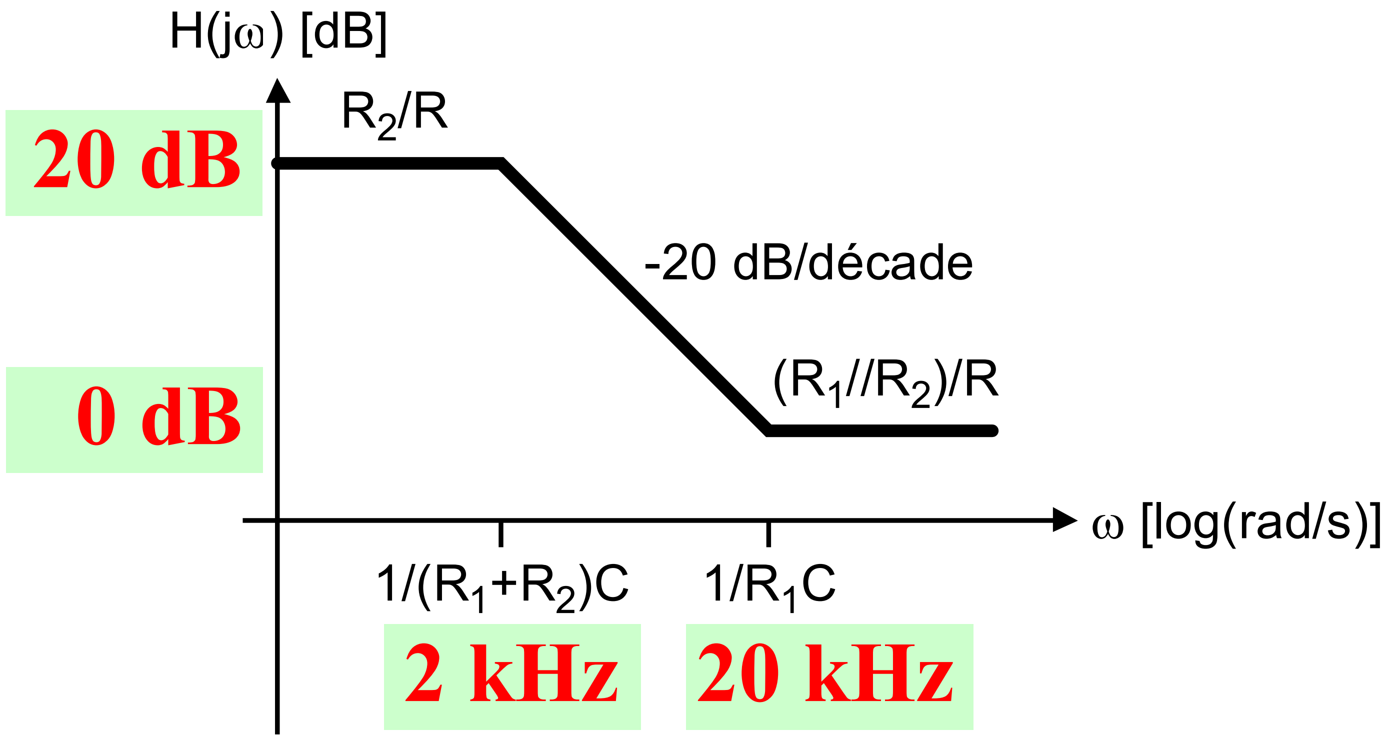




Equation de la fonction de transfert



$$\underline{H}(j\omega) = -\frac{\underline{Z}_2}{\underline{Z}_1} = -\frac{R_2}{R} \cdot \frac{1 + j\omega R_1 C}{1 + j\omega (R_1 + R_2) C}$$

Valeurs des composants

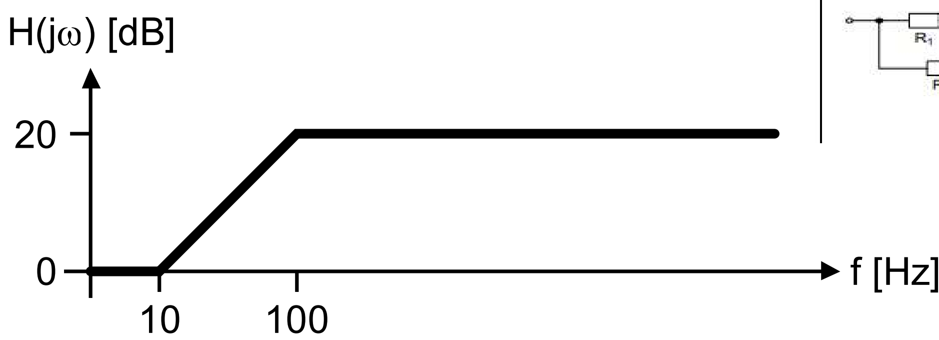
$$\frac{R_2}{R} = 10 \Leftrightarrow R = \frac{R_2}{10}$$

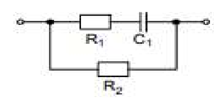
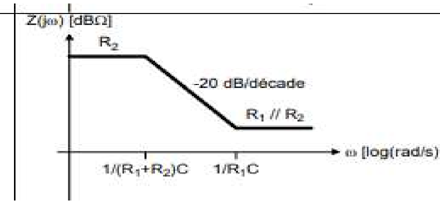
$$\frac{1}{(R_1 + R_2)C} = \frac{1}{10R_1C} \Leftrightarrow 10R_1 = R_1 + R_2 \Leftrightarrow R_1 = \frac{R_2}{9}$$

$$\frac{1}{R_1C} = 2\pi \cdot 20 \cdot 10^3 \Leftrightarrow C = \frac{1}{R_1 \cdot 2\pi \cdot 20 \cdot 10^3}$$

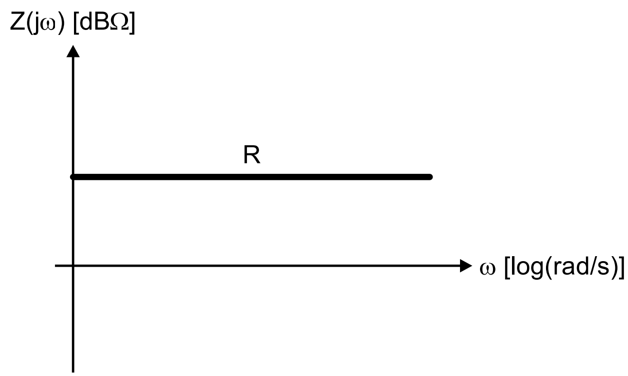
- On a un degré de liberté
  (libre choix de la valeur d'un composant)

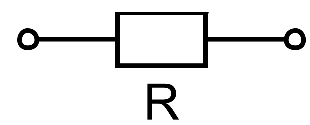
Example conception 2



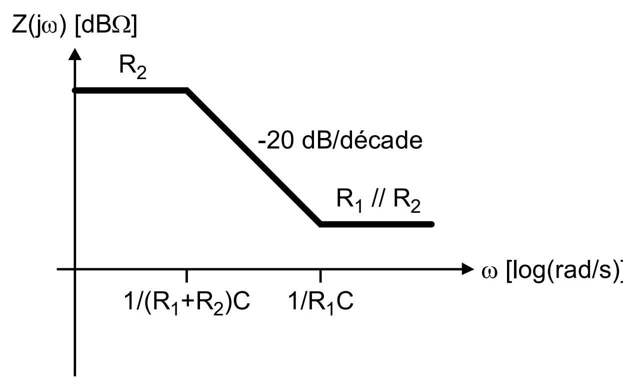
	$R_2 \dfrac{1+j\omega R_1 C}{1+j\omega(R_1+R_2)C}$	

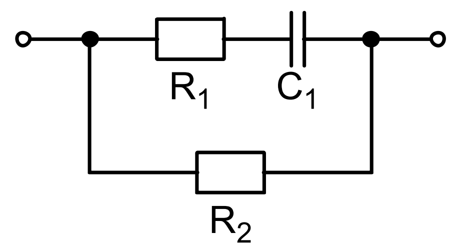
Choix des impédances et $\underline{H}(j\omega)$



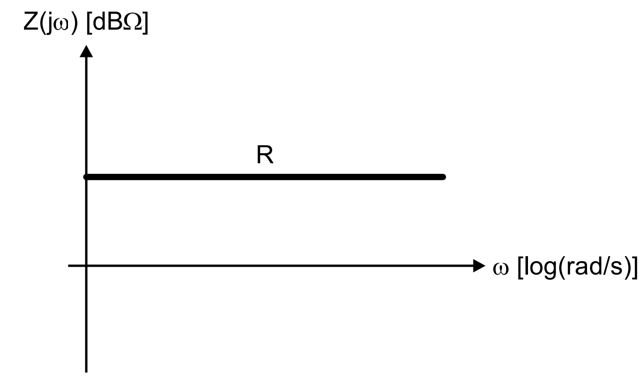


$$\left|\underline{H}(j\omega)\right| = \left|-\frac{\underline{Z}_2}{\underline{Z}_1}\right| = $$

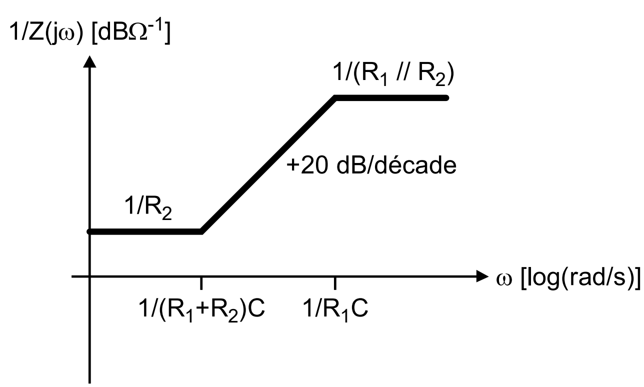




Fonction de transfert



× (+ en dB)



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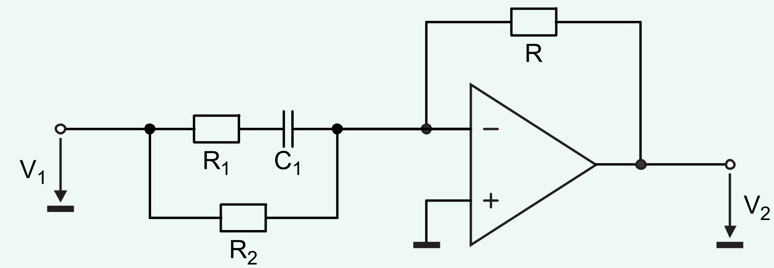
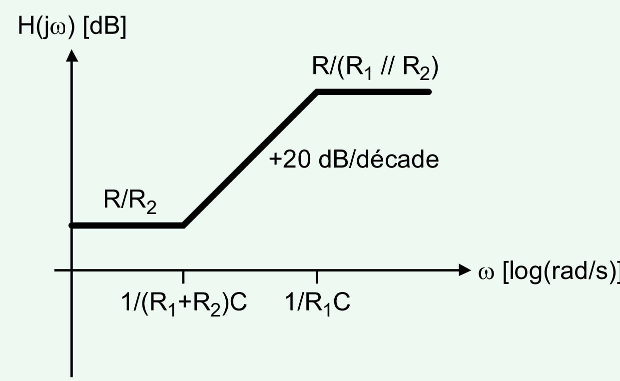
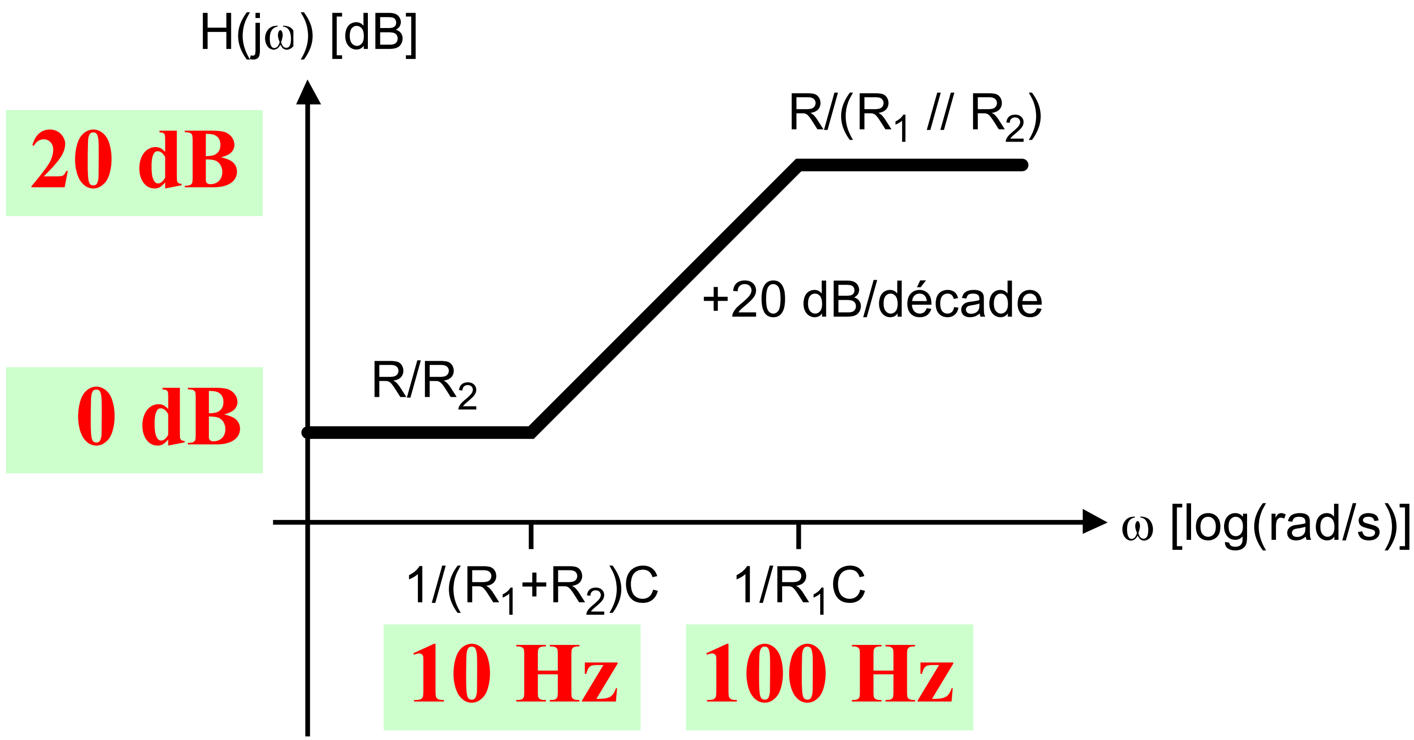




Equation de la fonction de transfert



$$\underline{H}(j\omega) = -\frac{\underline{Z}_2}{\underline{Z}_1} = -\frac{R}{R_2} \cdot \frac{1 + j\omega(R_1 + R_2)C}{1 + j\omega R_1 C}$$

Valeurs des composants

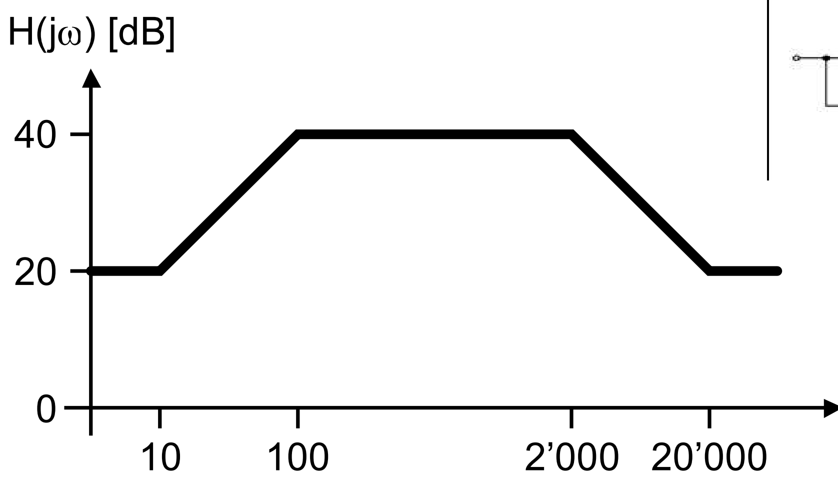
$$\frac{R}{R_2} = 1 \Leftrightarrow R = R_2$$

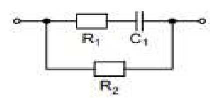
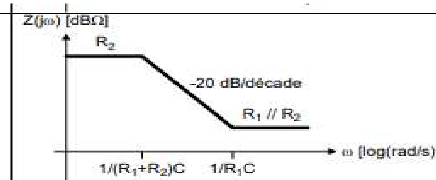
$$\frac{1}{(R_1 + R_2)C} = \frac{1}{10R_1C} \Leftrightarrow 10R_1 = R_1 + R_2 \Leftrightarrow R_1 = \frac{R_2}{9}$$

$$\frac{1}{R_1C} = 2\pi \cdot 100 \Leftrightarrow C = \frac{1}{R_1 \cdot 2\pi \cdot 100}$$

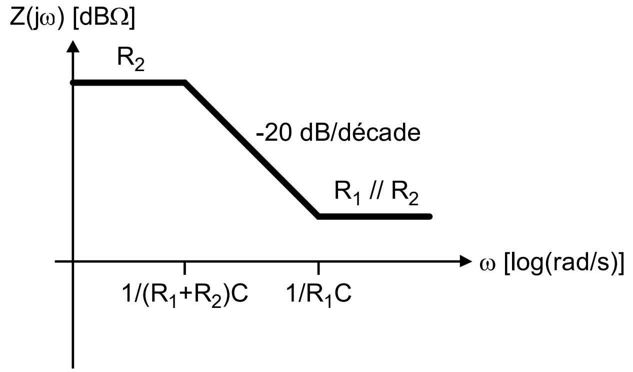
- On a un degré de liberté
  (libre choix de la valeur d'un composant)

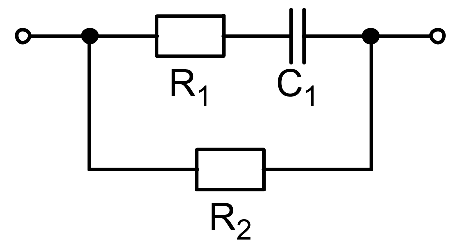
Exemple-conception 3:



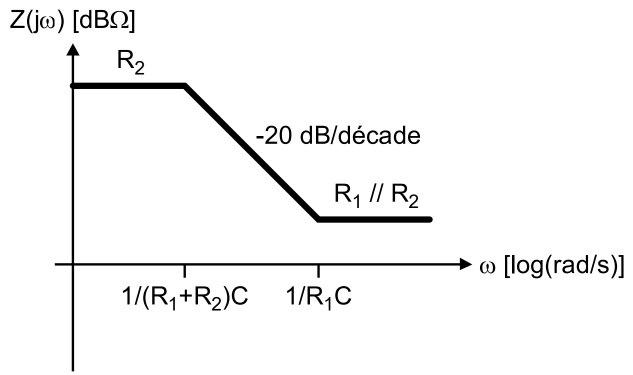
	$R_2 \dfrac{1+j\omega R_1 C}{1+j\omega (R_1+R_2) C}$	

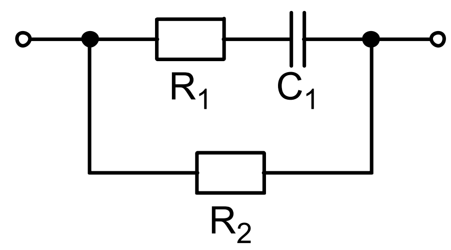
Choix des impédances et $\underline{H}(j\omega)$



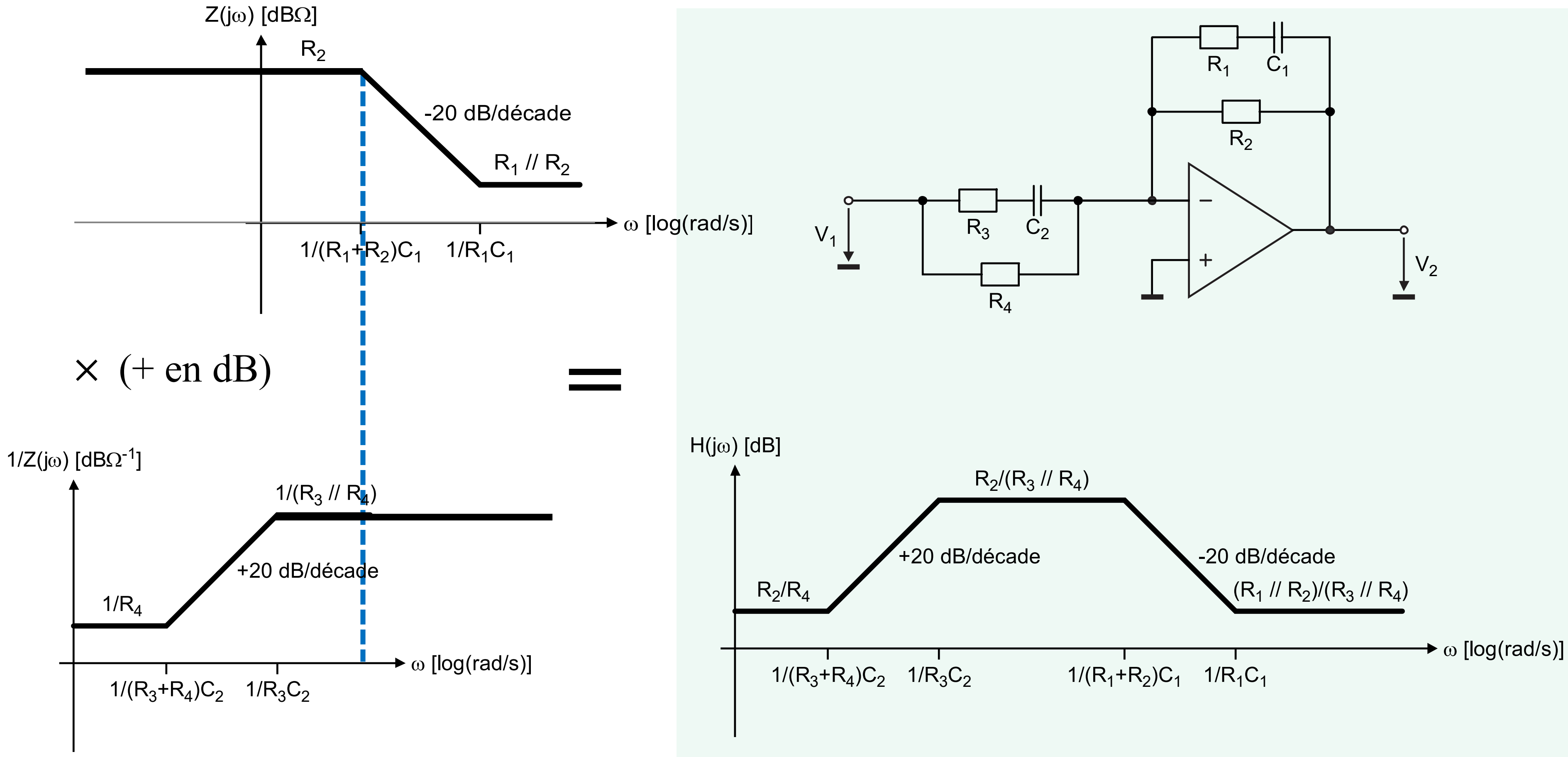


$$\left|\underline{H}(j\omega)\right| = \left|-\frac{\underline{Z}_2}{\underline{Z}_1}\right| =$$

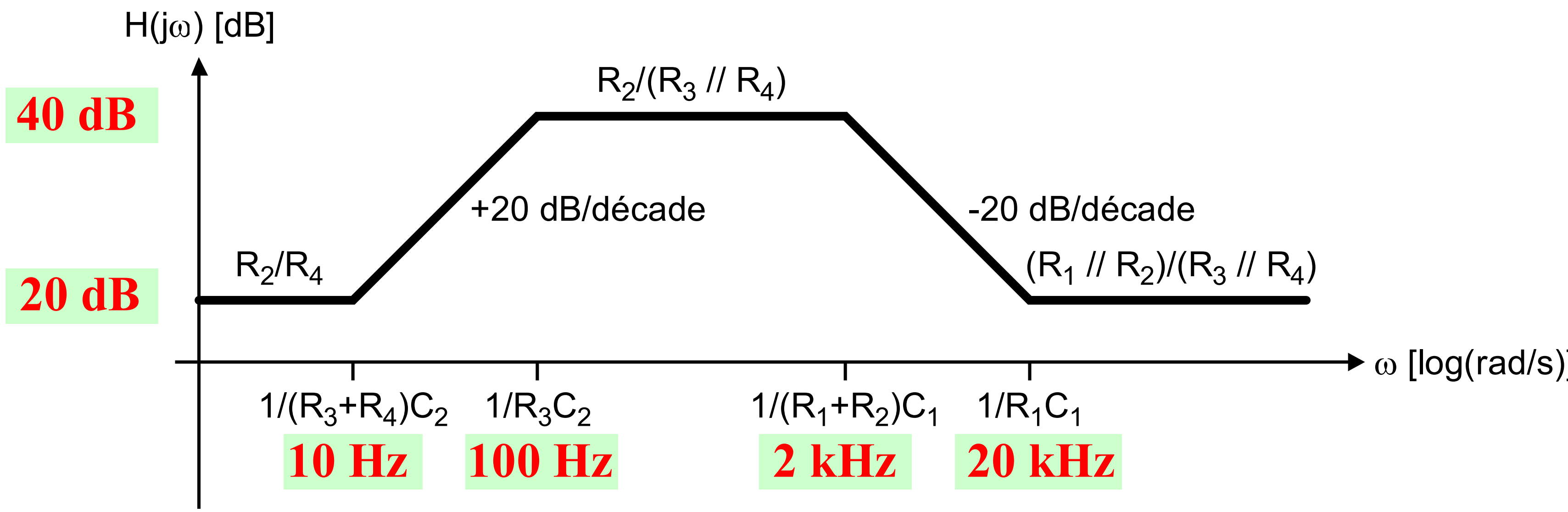




Fonction de transfert



Equation de la fonction de transfert



$$\underline{H}(j\omega) = -\frac{\underline{Z}_2}{\underline{Z}_1} = -\frac{R_2}{R_4} \cdot \frac{(1 + j\omega R_1 C_1) \cdot [1 + j\omega (R_3 + R_4) C_2]}{[1 + j\omega (R_1 + R_2) C_1] \cdot (1 + j\omega R_3 C_2)}$$

Valeurs des composants

$$\frac{R_2}{R_4} = 10 \Leftrightarrow R_2 = 10R_4$$

$$\frac{1}{(R_3 + R_4)C_2} = \frac{1}{10R_3C_2} \Leftrightarrow 10R_3 = R_3 + R_4 \Leftrightarrow R_3 = \frac{R_4}{9}$$

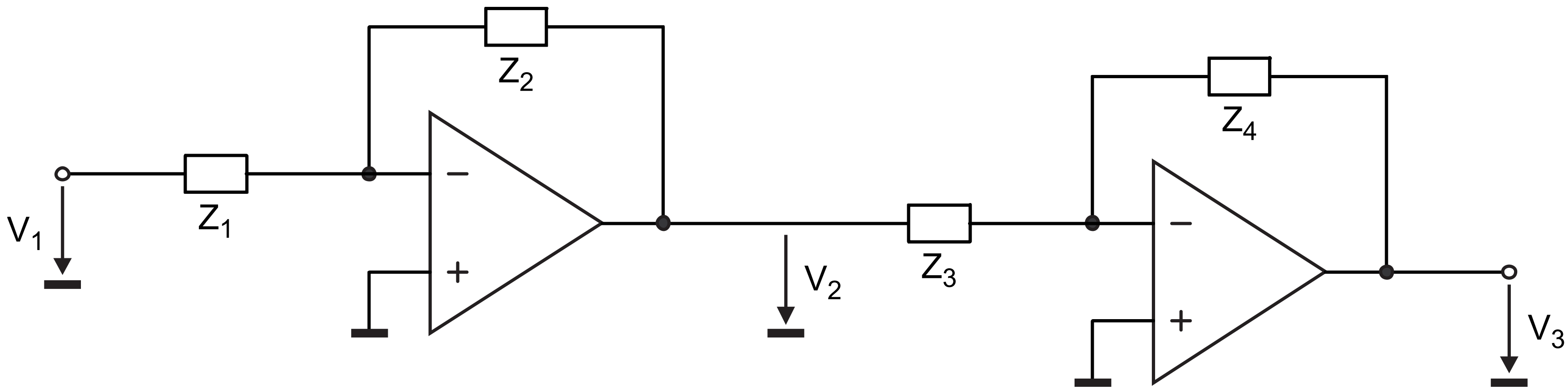
$$\frac{1}{(R_1 + R_2)C_1} = \frac{1}{10R_1C_1} \Leftrightarrow 10R_1 = R_1 + R_2 \Leftrightarrow R_1 = \frac{R_2}{9}$$

$$\frac{1}{R_3C_2} = 2\pi \cdot 100 \Leftrightarrow C_2 = \frac{1}{R_3 \cdot 2\pi \cdot 100}$$

$$\frac{1}{R_1C_1} = 2\pi \cdot 20 \cdot 10^3 \Leftrightarrow C_1 = \frac{1}{R_1 \cdot 2\pi \cdot 20 \cdot 10^3}$$

- On a un degré de liberté (libre choix de la valeur d’un composant)

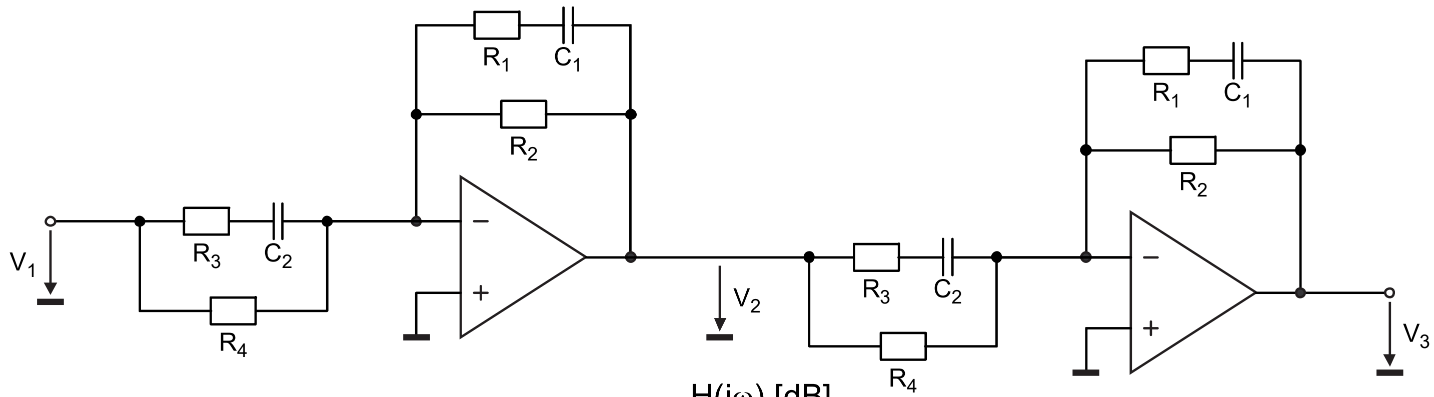
Filtres d'ordre > 1



$$\underline{H}(j\omega) = \frac{\underline{v}_3}{\underline{v}_1} = \frac{\underline{v}_2}{\underline{v}_1} \cdot \frac{\underline{v}_3}{\underline{v}_2} = \underline{H}_1(j\omega) \cdot \underline{H}_2(j\omega) = -\frac{\underline{Z}_2}{\underline{Z}_1} \cdot -\frac{\underline{Z}_4}{\underline{Z}_3}$$

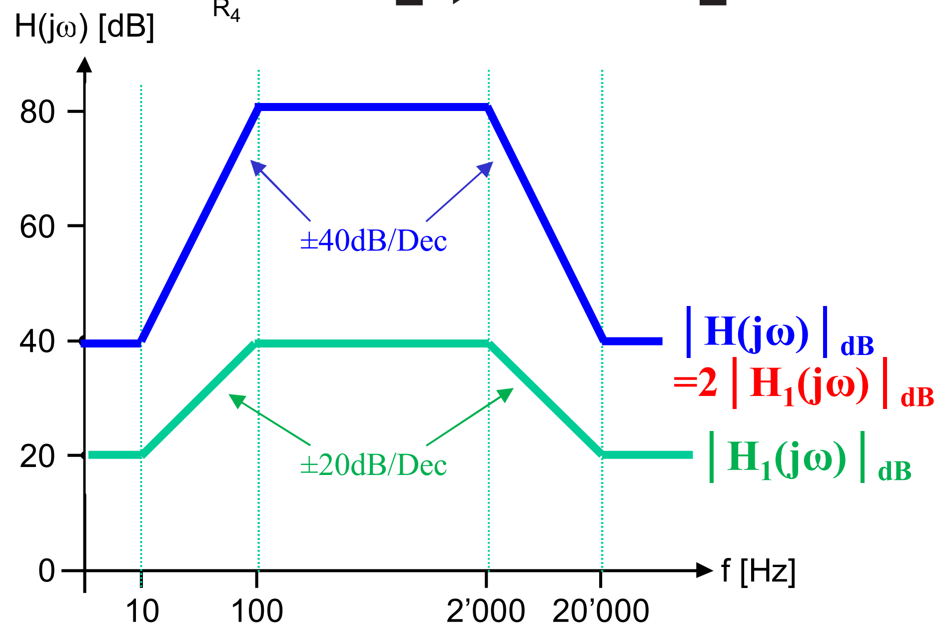
- Ordre n ≡ Pentes n x (± 20 dB/décade)
- Réalisé par mise en série de n filtres d'ordre 1

Exemple: Filtre d'ordre 2

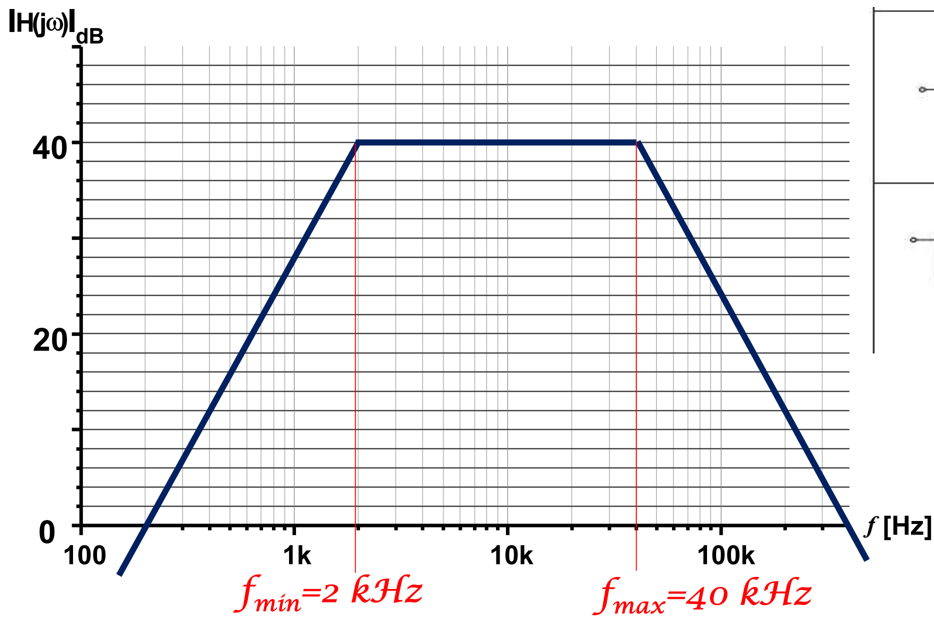


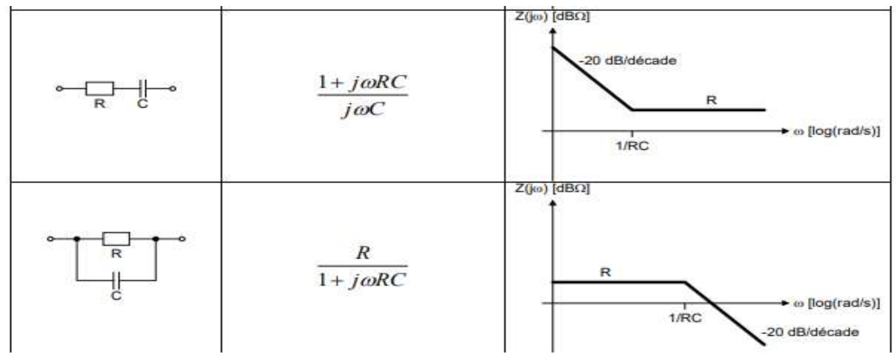
- 2 filtres passe-bande d'ordre 1 (exemple 3) en série

$$\underline{H}(j\omega) = \underline{H}_1(j\omega)\cdot\underline{H}_2(j\omega)$$
$$= \underline{H}_1^2(j\omega)$$

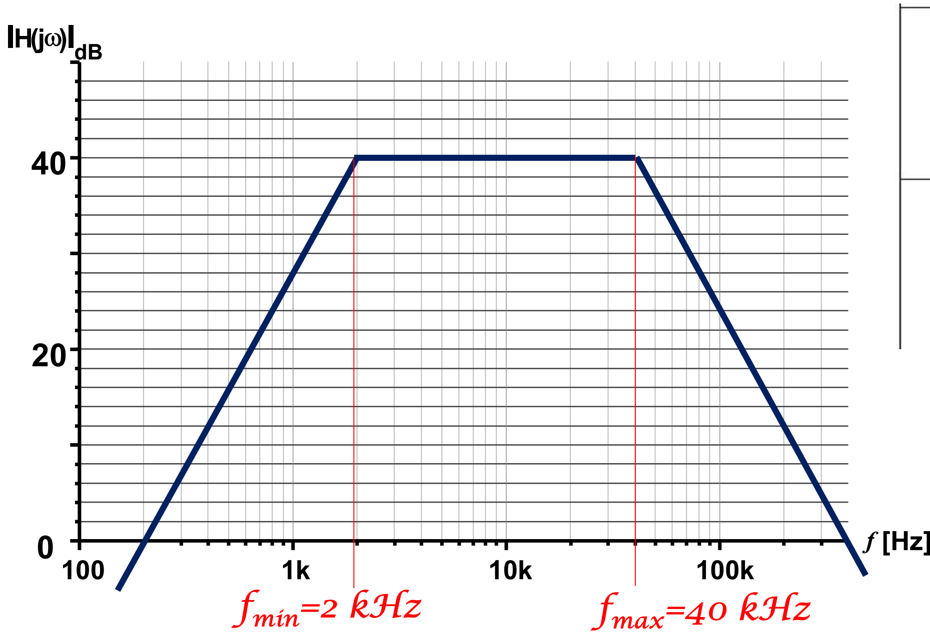


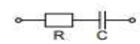
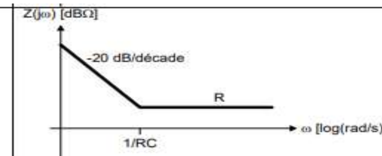
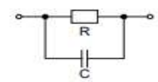
Ex: Filtrage





- **Donner l'architecture du filtre passe-bande à amplificateurs opérationnels réalisant la fonction de transfert ci-dessus.**
- **Dimensionner ses éléments : on prendra pour les résistances les plus faibles une valeur de 10 kΩ.**
- **Etablir l'expression analytique H(jω) correspondante, en mettant en évidence les pôles et les zéros.**
- **Donner le diagramme de Bode en phase**



	$\frac{1+j\omega RC}{j\omega C}$	
	$\frac{R}{1+j\omega RC}$